

The Born Rule Has Been Derived From Simple Physical Principles | Quanta Magazine

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[quantum physics](#)

The Born rule, which connects the math of quantum theory to the outcomes of experiments, has been derived from simpler physical principles. The new work promises to give researchers a better grip on the core mystery of quantum mechanics.



The Born rule, which converts quantum mechanical wave functions to precise predictions, was essentially a good guess.

Everyone knows that quantum mechanics is an odd theory, but they don't necessarily know why. The usual story is that it's the quantum world itself that's odd, with its superpositions, uncertainty and entanglement (the mysterious interdependence of observed particle states). All the theory does is reflect that innate peculiarity, right?

Not really. Quantum mechanics became a strange kind of theory not with Werner Heisenberg's famous uncertainty principle in 1927, nor when Albert Einstein and two colleagues identified (and Erwin Schrödinger named) entanglement in 1935. It happened in 1926, thanks to a proposal from the German physicist Max Born. Born suggested that the right way to interpret the wavy nature of quantum particles was as waves of probability. The wave equation presented by Schrödinger the previous year, Born said, was basically a piece of mathematical machinery for calculating the chances of observing a particular outcome in an experiment.

In other words, Born's rule connects quantum theory to experiment. It is what makes quantum mechanics a scientific theory at all, able to make predictions that can be tested. "The Born rule is the crucial link between the abstract mathematical objects of quantum theory and the world of experience," said [Lluís Masanes](#) of University College London.

The problem is that Born's rule was not really more than a smart guess — there was no fundamental reason that led Born to propose it. "It was an intuition without a precise justification," said [Adán Cabello](#), a quantum theorist at the University of Seville in Spain. "But it

worked.” And yet for the past 90 years and more, no one has been able to explain why.

Without that knowledge, it remains hard to figure out what quantum mechanics is telling us about the nature of reality. “Understanding the Born rule is important as a way to understand the picture of the world implicit in quantum theory,” said [Giulio Chiribella](#) of the University of Hong Kong, an expert on quantum foundations.

Several researchers have attempted to derive the Born rule from more fundamental principles, but none of those derivations have been widely accepted. Now Masanes and his collaborators [Thomas Galley](#) of the Perimeter Institute for Theoretical Physics in Waterloo, Canada, and [Markus Müller](#) of the Institute for Quantum Optics and Quantum Information in Vienna have [proposed](#) a new way to pull it out of deeper axioms about quantum theory, an approach that might explain how, more generally, quantum mechanics connects to experiment through the process of measurement.

“We derive all the properties of measurements in quantum theory: what the questions are, what the answers are, and what the probability of answers occurring are,” Masanes said.

It’s a bold claim. And given that the question of what measurement means in quantum mechanics has plagued the theory since the days of Einstein and Schrödinger, it seems unlikely that this will be the last word. But the approach of Masanes and colleagues is already winning praise. “I like it a lot,” Chiribella said.

The work “is a sort of ‘cleaning’ exercise,” Cabello said — a way of ridding quantum mechanics of redundant ingredients. “And that is absolutely an important task. These redundancies are a symptom that we don’t fully understand quantum theory.”

Where the Puzzle Is

Schrödinger wrote down his equation in 1925 as a formal description of the proposal by the French physicist Louis de Broglie the previous year that quantum particles such as electrons could behave like waves. The Schrödinger equation ascribes to a particle a wave function (denoted ψ) from which the particle’s future behavior can be predicted. The wave function is a purely mathematical expression, not directly related to anything observable.

The question, then, was how to connect it to properties that are observable. Schrödinger’s first inclination was to suppose that the amplitude of his wave function at some point in space — equivalent to the height of a water wave, say — corresponds to the density of the smeared-out quantum particle at that point.

But Born argued instead that the amplitude of the wave function is related to a probability — specifically, the probability that you will find the particle at that position if you detect it experimentally. In the lecture given for his 1954 Nobel Prize for this work, Born claimed that he had simply generalized from photons, the quantum “packets of light” that Einstein proposed in 1905. Einstein, Born said, had interpreted “the square of the optical wave amplitudes as probability density for the occurrence of photons. This concept could at once be carried over to the ψ -function.”

But this may have been a retrospective justification of a messier train of thought. For at first Born thought that it was simply the amplitude of ψ that gave this probability. He quickly decided

that it was the square of the wave function, ψ^2 (or, strictly speaking, the square of its modulus, or absolute value). But it was not immediately obvious which of these was right.

“Born got quantum theory to work using wire and bubble gum,” said [Mateus Araújo](#), a quantum theorist at the University of Cologne in Germany. “It’s ugly, we don’t really know why it works, but we know that if we take it out, the theory falls apart.”

Yet the arbitrariness of the Born rule is perhaps the least odd thing about it. In most physics equations, the variables refer to objective properties of the system they are describing: the mass or velocity of bodies in Newton’s laws of motion, for instance. But according to Born, the wave function is not like this. It’s not obvious whether it says anything about the quantum entity itself — such as where it is at any moment in time. Rather, it tells us what we might see if we choose to look. It points in the wrong direction: not down toward the system being studied, but up toward the observer’s experience of it.

“What makes quantum theory puzzling is not so much the Born rule as a way of computing probabilities,” Chiribella said, “but the fact that we cannot interpret the measurements as revealing some pre-existing properties of the system.”

What’s more, the mathematical machinery for unfolding these probabilities can only be written down if you stipulate *how* you’re looking. If you do different measurements, you might calculate different probabilities, even though you seem to be examining the same system in both cases.

That’s why Born’s prescription for turning wave functions into measurement outcomes contains all of the reputed paradoxical nature of quantum theory: the fact that observable properties of quantum objects emerge, in a probabilistic way, from the act of measurement itself. “Born’s probability postulate is where the puzzle really is,” Cabello said.

So if we could understand where the Born rule comes from, we might finally understand what the vexed concept of measurement really means in quantum theory.

The Argument

That’s what has largely motivated efforts to explain the Born rule — rather than simply to learn and accept it. One of the most celebrated attempts, presented by the American mathematician Andrew Gleason in 1957, shows that the rule follows from some of the other components of the standard mathematical structure of quantum mechanics: In other words, it’s a tighter package than it originally seemed. All the same, Gleason’s approach assumes some key aspects of the mathematical formalism needed to connect quantum states to specific measurement outcomes.

One very different approach to deriving the Born rule draws on the controversial many-worlds interpretation of quantum mechanics. Many-worlds is an attempt to solve the puzzle of quantum measurements by assuming that, instead of selecting just one of the multiple possible outcomes, an observation realizes all of them — in different universes that split off from our own. In the late 1990s, many-worlds advocate [David Deutsch](#) asserted that apparent quantum probabilities are precisely what a rational observer would need to use to make predictions in such a scenario — an argument that can be used to derive the Born rule. Meanwhile, [Lev Vaidman](#) of Tel Aviv University in Israel, and independently [Sean Carroll](#) and [Charles Sebens](#) of the California Institute of Technology, [suggested](#) that the Born rule is the only one that assigns correct probabilities in a many-worlds multiverse during the instant after a split has occurred but

before any observers have registered the outcome of the measurement. In that instant the observers do not yet know which branch of the universe they are on — but Carroll and Sebens argued that “there is a uniquely rational way to apportion credence in such cases, which leads directly to the Born Rule.”

The [many-worlds picture leads to its own problems](#), however — not least the issue of what “probability” can mean at all if every possible outcome is definitely realized. The many-worlds interpretation “requires a radical overhaul of many fundamental concepts and intuitions,” Galley said. What’s more, some say that there is no coherent way to connect an observer before a split to the same individual afterward, and so it is logically unclear what it means for an observer to apply the Born rule to make a prediction “before the event.” For such reasons, many-worlds derivations of the Born rule are not widely accepted.

Masanés and colleagues have now [set out an argument](#) that does not require Gleason’s assumptions, let alone many universes, to derive the Born rule. While the rule is typically presented as an add-on to the basic postulates of quantum mechanics, they show that the Born rule follows from those postulates themselves once you admit that measurements generate unique outcomes. That is, if you grant the existence of quantum states, along with the “classical” experience that just one of them is actually observed, you’ve no choice but to square the wave function to connect the two. “Our result shows that not only is the Born rule a good guess, but it is the only logically consistent guess,” Masanés said.

To reach that conclusion, we just need a few basic assumptions. The first is that quantum states are formulated in the usual way: as vectors, possessing both a size and a direction. It’s not that different from saying that each place on Earth can be represented as a point assigned a longitude, latitude and altitude.

The next assumption is also a completely standard one in quantum mechanics: So long as no measurement is made on a particle, it changes in time in a way that is said to be “unitary.” Crudely speaking, this means that the changes are smooth and wavelike, and they preserve information about the particle. This is exactly the behavior that the Schrödinger equation prescribes, and it is in fact unitarity that makes measurement such a headache — because measurement is a non-unitary process, often dubbed the “collapse” of the wave function. In a measurement, only one of several potential states is observed: Information is lost.

The researchers also assume that, for a system of several parts, how you group those parts should make no difference to a measurement outcome. “This assumption is so basic that it is in some sense a precondition of any reasoning about the world,” Galley said. Suppose you have three apples. “If I say, ‘There are two apples on the right and one on the left,’ and you say, ‘There are two apples on the left and one on the right,’ then these are both valid ways of describing the apples. The fact of where we place the dividing line of left and right is a subjective choice, and these two descriptions are equally correct.”

The final assumption embraces measurement itself — but in the most minimal sense conceivable. Simply, a given measurement on a quantum system *must* produce a unique outcome. There’s no assumption about *how* that happens: how the quantum formalism must be used to predict the probabilities of the outcomes. Yet the researchers show that this process has to follow the Born rule if the postulate about uniqueness of measurement is to be satisfied. Any alternatives to the Born rule for deriving probabilities of observed outcomes from the wave function won’t satisfy the initial postulates.

The result goes further than this: It could also clear up what the measurement machinery of quantum mechanics is all about. In short, there's a whole technical paraphernalia of requirements in that mechanism: mathematical functions called Hermitian operators that "operate on" the wave function to produce things called eigenvalues that correspond to measurement probabilities, and so on. But none of that is assumed from the outset by Masanes and colleagues. Rather, they find that, like the Born rule, all of these requirements are implicit in the basic assumptions and aren't needed as extras.

"We just assume that there are questions, and when asked these return a single answer with some probability," Galley said. "We then take the formalism of quantum theory and show that the only questions, answers and probabilities are the quantum ones."

The work can't answer the troublesome question of why measurement outcomes are unique; rather, it makes that uniqueness axiomatic, turning it into part of the very definition of a measurement. After all, Galley said, uniqueness "is required for us to be able to even begin to do science."

However, what qualifies as a "minimal" assumption in quantum theory is rarely if ever straightforward. Araújo thinks that there may be more lurking in these assumptions than meets the eye. "They go far beyond assuming that a measurement exists and has a unique outcome," he said. "Their most important assumption is that there is a fixed set of measurements whose probabilities are enough to completely determine a quantum state." In other words, it's not just a matter of saying measurements exist, but of saying that measurements — with corresponding probabilities of outcomes — are able to tell you everything you can know. That might sound reasonable, but it is not self-evidently true. In quantum theory, few things are.

So while Araújo calls the paper "great work," he adds, "I don't think it really explains the Born rule, though, any more than noticing that without water we die explains what water is." And it leaves hanging another question: Why does the Born rule only specify probabilities, and not definite outcomes?

Law Without Law

The project pursued here is one that has become popular with several researchers exploring the foundations of quantum mechanics: to see whether this seemingly exotic but rather ad hoc theory can be derived from some simple assumptions that are easier to intuit. It's a program called [quantum reconstruction](#).

Cabello has pursued that aim too, and has [suggested an explanation of the Born rule](#) that is similar in spirit but different in detail. "I am obsessed with finding the simplest picture of the world that enforces quantum theory," he said.

His approach starts with the challenging idea that there is in fact no underlying physical law that dictates measurement outcomes: Every outcome may take place so long as it does not violate a set of logical-consistency requirements that connect the outcome probabilities of different experiments. For example, let's say that one experiment produces three possible outcomes (with particular probabilities), and a second independent experiment produces four possible outcomes. The combined number of possible outcomes for the two experiments is three times four, or 12 possible outcomes, which form a particular, mathematically defined set of combined possibilities.

Such a lawless reality sounds like an unlikely recipe for producing a quantitatively predictive theory like quantum mechanics. But in 1983 the American physicist John Wheeler [proposed](#) that statistical regularities in the physical world might emerge from such a situation, as they sometimes do from unplanned crowd behavior. “Everything is built higgledy-piggledy on the unpredictable outcomes of billions upon billions of elementary quantum phenomena,” Wheeler wrote. But there might be no fundamental law governing those phenomena — indeed, he argued, that was the only scenario in which we could hope to find a self-contained physical explanation, because otherwise we’re left with an infinite regression in which any fundamental equation governing behavior needs to be accounted for by some even more fundamental principle. “In contrast to the view that the universe is a machine governed by some magic equation, ... the world is a self-synthesizing system,” Wheeler argued. He called this emergence of the lawlike behavior of physics “law without law.”

Cabello finds that, if measurement outcomes are constrained to obey the behaviors seen in quantum systems — where for example certain measurements can be correlated in ways that make them interdependent (entangled) — they must also be prescribed by the Born rule, even in the absence of any deeper law that dictates them.

“The Born rule turns out to be a logical constraint that should be satisfied by any reasonable theory we humans can construct for assigning probabilities when there is no law in the physical reality governing the outcomes,” Cabello said. The Born rule is then dictated merely by logic, not by any underlying physical law. “It has to be satisfied the same way as the rule that the probabilities must be between 0 and 1,” Cabello said. The Born rule itself, he said, is thus an example of Wheeler’s “law without law.”

But is it really that? Araújo thinks that Cabello’s approach doesn’t sufficiently explain the Born rule. Rather, it offers a rationale for which quantum correlations (such as those seen in entanglement) are allowed. And it doesn’t eliminate all possible laws governing them, but only those that are forbidden by the consistency principles. “Once you’ve determined which [correlations] are the forbidden ones, everything that remains is allowed,” Araújo said. So it could be lawless down there in the quantum world — or there could be some other self-consistent but still law-bound principle behind what we see.

Any Possible Universe

Although the two studies pull out the Born rule from different origins, the results are not necessarily inconsistent, Cabello said: “We simply have different obsessions.” Masanes and colleagues are looking for the simplest set of axioms for constructing the operational procedures of quantum mechanics — and they find that, if measurement as we know it is possible at all, then the Born rule doesn’t need to be added in separately. There’s no specification of what kind of underlying physical reality gives rise to these axioms. But that underlying reality is exactly where Cabello starts from. “In my opinion, the really important task is figuring out which are the physical ingredients common to any universe in which quantum theory holds,” he said. And if he’s right, those ingredients lack any deep laws.

Evidently that remains to be seen: Neither of these papers will settle the matter. But what both studies have in common is that they aim to show how at least some of the recondite, highly mathematical and apparently rather arbitrary quantum formalism can be replaced with simple postulates about what the world is like. Instead of saying that “probabilities of measurement outcomes are equal to the modulus squared of the wave function,” or that “observables

correspond to eigenvalues of Hermitian operators,” it’s enough to say that “measurements are unique” or that “no fundamental law governs outcomes.” It might not make quantum mechanics seem any less strange to us, but it could give us a better chance of understanding it.